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Semi-inclusive B Decays and Direct CP Violation in QCD Factorization

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Abstract

We have systematically investigated the semi-inclusive B decays $B \to MX$, which are manifestations of the quark decay $b \to Mq$, within the framework of QCD-improved factorization. These decays are theoretically clean and have distinctive experimental signatures. We focus on a class of these that do not require any form factor information and therefore may be especially suitable for extracting information on the angles α and γ of the unitarity triangle. The nonfactorizable effects, such as vertex-type and penguin-type corrections to the two-body b decay, $b \to Mq$, and hard spectator corrections to the 3-body decay $B \to Mq_1\bar{q}_2$ are calculable in the heavy quark limit. QCD factorization is applicable when the emitted meson is a light meson or a charmonium. We discuss the issue of the CPT constraint on partial rate asymmetries. The strong phase coming from final-state rescattering due to hard gluon exchange between the final states can induce large rate asymmetries for tree-dominated color-suppressed modes $(\pi^0, \rho^0, \omega)X_{\bar{s}}$. The nonfactorizable hard spectator interactions in the 3-body decay $B \to Mq_1\bar{q}_2$, though phase-space suppressed, are extremely important for the tree-dominated modes $(\pi^0, \rho^0, \omega)X_{\bar{s}}$, ϕX, $J/\psi X_s, J/\psi X$ and the penguin-dominated mode $\omega X_{s\bar{s}}$. In fact, they are dominated by the hard spectator corrections. This is because the relevant hard spectator interaction is color allowed, whereas the two-body semi-inclusive decays for these modes are color-suppressed. Our result for $\mathcal{B}(B \to J/\psi X_s)$ is in agreement with experiment. The semi-inclusive decay modes: $\overline{B}{}_s^0 \to (\pi^0, \rho^0, \omega)X_{\bar{s}}$, $\rho^0 X_{s\bar{s}}$, $\overline{B}{}^0 \to (K^- X, K^{*-}X)$ and $B^- \to (K^0 X_s, K^{*0} X_s)$ are the most promising ones in searching for direct CP violation. In fact, they have branching ratios of order $10^{-6} - 10^{-4}$ and CP rate asymmetries of order $(10-40)\%$. The decays $\overline{B}{}_s^0 \to (\pi^0, \rho^0, \omega)X_{s\bar{s}}$ and $B^- \to \phi X$ are electroweak-penguin dominated. Some of them have sizable branching ratios and observable CP asymmetries.

I. INTRODUCTION

Recently Beneke, Buchalla, Neubert and Sachrajda (BBNS) [1] have proposed a QCD-improved factorization approach to a class of exclusive B-decays that appears quite promising. BBNS suggest that nonfactorizable effects in an exclusive decay $B \to M_1 M_2$ with recoiled M_1 and emitted light meson M_2 are calculable since only hard interactions between the (BM_1) system and M_2 survive in the heavy quark limit. In this paper we extend the application of BBNS idea of QCD factorization to a certain class of semi-inclusive decays. In this regard our approach complements the recent works of He *et al* [2,3]. The semi-inclusive decays that are of special interest originate from the quark level decay, $b \to M + q$; they are theoretically cleaner compared to exclusive decays and have distinctive experimental signatures [4]. Since these semi-inclusive decays also tend to have appreciably larger branching ratios compared to their exclusive counterparts, they may therefore be better suited for extracting CKM-angles and for testing the Standard Model (SM).

Earlier studies of semi-inclusive decays are based on naive factorization [4] or generalized factorization [5] in which nonfactorizable effects are treated in a phenomenological way by assuming that, for example, the number of colors is a free parameter to be fitted to the data. Apart from the unknown nonfactorizable corrections, this approach encounters another major theoretical uncertainty, namely the gluon's virtuality k^2 in the penguin diagram is basically unknown, rendering the predictions of CP asymmetries not trustworthy.

The aforementioned difficulties with the conventional methods can be circumvented in the BBNS approach of QCD-improved factorization (QCDF). Indeed, by placing an energy cut $E_M \geq 2.1$ GeV on the meson in the semi-inclusive signal $B \to M + X$, not only we can enhance the presence of the two body quark decays, $b \to M + q$, but also M then can play the role of M_2 and X that of the recoiled meson M_1 in the above-mentioned exclusive decay $B \to M_1 + M_2$, in so far as considerations of BBNS go. Furthermore, a very important theoretical simplification occurs in the semi-inclusive decays over the exclusive decays if we focus on final states such that M does not contain the spectator quark of the decaying $B(B_s)$ meson as then we completely by-pass the need for the transition form factor for $B(B_s) \to M$. Recall that for the exclusive case, in general, we need a knowledge of two such form factors if M is a pseudoscalar meson or of four form factors if M is a vector meson.

The consideration of these semi-inclusive B decays has several other theoretical advantages over the exclusive ones as well. For one thing, there is no troublesome infrared divergent problem occurred at endpoints when working in QCD factorization. As for CP violation, contrary to the exclusive hadronic decays, it is not plagued by the unknown soft phases. Consequently, the predictions of the branching ratios and partial rate asymmetries for $B \to MX$ are considerably clean and reliable.

Recently QCD factorization has been applied to charmless semi-inclusive decays $B \to$

$K(K^*)X$ and $B \to \phi X_s$ in [2,3]. In the present paper we will systematically study all semi-inclusive decays $B \to PX(X_s, X_c)$, $B \to VX(X_s, X_c)$ and $B \to J/\psi X$ with P (V) being a light pseudoscalar (vector) meson, X the final state containing no charmed or strange particles and X_s the final state containing a strange quark and X_c the final state containing a charm quark. We differ from [2,3] in two main aspects: First, we have included the complete twist-3 corrections to the penguin coefficient a_6 to be introduced below and electroweak penguin-type corrections to the coefficients a_{7-10} induced by tree 4-quark operators; both have been neglected in [2,3]. Second, the hard spectator interaction in the 3-body decay $B \to Mq_1\bar{q}_2$ is neglected in [2,3], whereas we will show that it is quite important for color-suppressed modes. As a by-product, we shall see that the troublesome infrared divergent problem encountered in the exclusive two-body decays does not occur in the semi-inclusive case.

The consideration of semi-inclusive decays $B \to MX$ has two more complications than the corresponding two-body decay, $b \to Mq$. First, in the free quark approximation, the fact that $B \to MX$ can be viewed as the free b quark decay $b \to Mq$ is justified only in the heavy quark limit. For the finite b quark mass, it becomes necessary to consider the initial b quark bound state effect. This has been investigated recently in [2] using two different approaches which we will follow closely. Second, the 3-body decay $\overline{B} \to M + q_1 + \bar{q}_2$ with the quark content $(b\bar{q}_2)$ for the $\overline{B}$ meson could be important for color-suppressed modes as just mentioned before. Here one needs a hard gluon exchange between the spectator quark $\bar{q}_2$ and the meson M in order to ensure that the outgoing $\bar{q}_2$ is hard.

In passing, we briefly recall that for experimental purposes a useful feature of these semi-inclusive decays, $B \to M + X$, with an energetic M, is that these events should have relatively low multiplicity [4].

The paper is organized as follows. In Sec. II we will outline the QCD factorization approach relevant for our purposes and then we proceed to apply it to the two-body decays of the b quark $b \to Mq$ in Sec. III. In Sec. IV we study the hard spectator corrections and summarize the results for branching ratios and CP rate asymmetries. Sec. V gives our conclusions.

II. QCD FACTORIZATION

In this section we want to suggest that the idea of QCD factorization [1] can be extended to the case of semi-inclusive decays, $B \to M + X$, with rather energetic meson M, say $E_M \geq 2.1$ GeV. Recall that it has been shown explicitly [1] that if the emitted meson M_2 is a light meson or a quarkonium in the two-body exclusive decay $B \to M_1M_2$ with M_1 being a recoiled meson, the transition matrix element of an operator O, namely $\langle M_1M_2|O|B\rangle$, is factorizable in the heavy quark limit. Schematically one has [1]

$$
\begin{aligned}
\langle M_1 M_2|O_i|B\rangle &= \langle M_1 M_2|O_i|B\rangle_{\rm fact}\left[1+\sum r_n\alpha_s^n+\mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})\right] \\
&= \sum_j F_j^{BM_1}(m_2^2)\int_0^1 du\, T_{ij}^I(u)\Phi_{M_2}(u) \\
&\quad + \int_0^1 d\xi\, du\, dv\, T_i^{II}(\xi,u,v)\Phi_B(\xi)\Phi_{M_1}(u)\Phi_{M_2}(v),
\end{aligned} \tag{2.1}
$$

where F^{BM_1} is a $B-M_1$ transition form factor, Φ_M is the light-cone distribution amplitude, and T^I, T^{II} are perturbatively calculable hard scattering kernels. In the naive factorization approach, T^I is independent of u as it is nothing but the meson decay constant. However, large momentum transfer to M_2 conveyed by hard gluon exchange implies a nontrivial convolution with the distribution amplitude Φ_{M_2}. The second hard scattering function T^{II}, which describes hard spectator interactions, survives in the heavy quark limit when both M_1 and M_2 are light or when M_1 is light and M_2 is a quarkonium [1]. The factorization formula (2.1) implies that naive factorization is recovered in the $m_b\to\infty$ limit and in the absence of QCD corrections. Nonfactorizable corrections are calculable since only hard interactions between the (BM_1) system and M_2 survive in the heavy quark limit.

As an illustrative example of a case when QCDF is not applicable, let us mention the decay $\overline{B}^0\to\pi^0 D^0$. QCDF does not work here because the emitted meson D^0 is heavy so that it is neither small (with size of order $1/\Lambda_{\rm QCD}$) nor fast and cannot be decoupled from the $(B\pi)$ system. This is also ascribed to the fact that the soft interaction between $(B\pi)$ and the c quark of the D^0 meson is not compensated by that between $(B\pi)$ and the light spectator quark of the charmed meson.

As for the semi-inclusive decay $B\to MX$, a momentum cutoff imposed on the emitted light meson M, say $p_M>2.1$ GeV, is necessary in order to reduce contamination from the unwanted background and ensure the relevance of the two-body quark decay $b\to Mq$. For example, an excess of $K(K^*)$ production in the high momentum region, $2.1<p_{K(K^*)}<2.7$ GeV, will ensure that the decay $B\to K(K^*)X$ is not contaminated by the background $b\to c$ transitions manifested as $B\to D(D^*)X\to K(K^*)X'$ and that it is dominated by the quasi two-body decay $b\to K(K^*)q$ induced from the penguin process $b\to sg^*\to sq\bar{q}$ and the tree process $b\to u\bar{u}s$. As we shall see shortly, the kinematic consideration here will restrict the possible forms of factorization for the matrix element $\langle MX|O|B\rangle$. By the same physical argument as in the exclusive case and by treating $M_1=X$ and $M_2=M$, it is natural to expect that the factorization formula (2.1) can be generalized to the semi-inclusive decay:

$$
\begin{aligned}
\langle MX|O|B\rangle &= \langle MX|O|B\rangle_{\rm fact}\left[1+\sum r_n\alpha_s^n+\mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})\right] \\
&= \int_0^1 du\, T^I(u)\Phi_M(u)+\int_0^1 d\xi\, du\, T^{II}(\xi,u)\Phi_B(\xi)\Phi_M(u).
\end{aligned} \tag{2.2}
$$

In comparing this equation to the exclusive case Eq. (2.1), a crucial simplification that has occurred is that the semi-inclusive case does not involve any transition form factor(s).

This attractive feature holds so long as the meson M does not contain the spectator quark in the initial B meson. Since lack of knowledge of these form factors is often a serious limitation in quantitative applications, this adds to the appeal of the semi-inclusive case. Note also that when the emitted meson M is a light meson or a quarkonium, the nonfactorizable corrections to naive factorization are infrared safe in the heavy quark limit and hence calculable. However, by the same token as the $\overline{B}^0 \to \pi^0 D^0$ decay, the above QCD factorization formula is also not applicable to $\overline{B}^0 \to D^0(\overline{D}^0)X$. The analog of $\overline{B}^0 \to D^+\pi^-$ in semi-inclusive decays is $\overline{B}^0 \to \pi^- X_c$. However, there is one difference between them, namely the hard spectator interaction proportional to the kernel T^{II} vanishes in the decay $\overline{B}^0 \to D^+\pi^-$, while it survives in the 3-body decay $\overline{B}^0 \to \pi^- c\,\bar{q}$, where $\bar{q}$ is the spectator quark of the B meson. This is because the spectator quark in the B and D bound states in $B-D$ transition is very soft in the heavy quark limit, whereas the same quark $\bar{q}$ appearing in the 3-body decay has to be hard so that a hard gluon exchange between π and $\bar{q}$ is needed.

The factorizable hadronic matrix element $\langle MX|O|B\rangle$ has the general expression:

$$\langle MX|O|B\rangle_{\rm fact} = \langle M|j_1|0\rangle\langle X|j_2|B\rangle + \langle X|j_1'|0\rangle\langle M|j_2'|B\rangle, \tag{2.3}$$

where j_1' and j_2' arise from the Fierz transformation of the operator O and the annihilation term $\langle MX|j_1|0\rangle\langle 0|j_2|B\rangle$ is suppressed by order $\Lambda_{\rm QCD}/m_b$ and hence it will not be included in Eq. (2.3). As stressed in [2], Eq. (2.3) is not the only way for factorization; there are other ways of factorization, for example, $\langle X_1M|j_1|0\rangle\langle X_1'|j_2|B\rangle$ with $X_1 + X_1' = X$. For the aforementioned quasi-two-body decay, it seems quite plausible to expect that the configuration $\langle X_1M|j_1|0\rangle\langle X_1'|j_2|B\rangle$ is dominated by $\langle M|j_1|0\rangle\langle X|j_2|B\rangle$ [2]; at least in the perturbative region, the production of additional X_1 in the final state is suppressed by powers of α_s [2].

The effective Hamiltonian relevant for hadronic semi-inclusive B decays of interest has the form

$$\begin{aligned}\mathcal{H}_{\rm eff}(\Delta B=1) = \frac{G_F}{\sqrt{2}}\Bigg\{& V_{ub}V_{uq}^*\Big[c_1(\mu)O_1^u(\mu)+c_2(\mu)O_2^u(\mu)\Big] + V_{cb}V_{cq}^*\Big[c_1(\mu)O_1^c(\mu)+c_2(\mu)O_2^c(\mu)\Big] \\ & -V_{tb}V_{tq}^*\left(\sum_{i=3}^{10}c_i(\mu)O_i(\mu)+c_g(\mu)O_g(\mu)\right) \\ & +V_{cb}V_{uq}^*\Big[c_1(\mu)\tilde{O}_1(\mu)+c_2(\mu)\tilde{O}_2(\mu)\Big]\Bigg\} + {\rm h.c.},\end{aligned} \tag{2.4}$$

where $q = d, s$ and

$$\begin{aligned} O_1^u &= (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, & O_2^u &= (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \\ O_1^c &= (\bar{c}b)_{V-A}(\bar{q}c)_{V-A}, & O_2^c &= (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta c_\alpha)_{V-A}, \\ \tilde{O}_1 &= (\bar{c}b)_{V-A}(\bar{q}u)_{V-A}, & \tilde{O}_2 &= (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \end{aligned}$$

$$
\begin{aligned}
&O_{3(5)} = (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A(V+A)}, && O_{4(6)} = (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \\
&O_{7(9)} = \frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V+A(V-A)}, && O_{8(10)} = \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}, \\
&O_g = \frac{g_s}{8\pi^2}\, m_b\bar{q}\sigma^{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}(1+\gamma_5)b,
\end{aligned}
\tag{2.5}
$$

with $q' = u, d, s$, $(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$, O_3–O_6 being the QCD penguin operators, O_7–O_{10} the electroweak penguin operators, and O_g the chromomagnetic dipole operator. After the inclusion of vertex-type and penguin-type corrections, we obtain

$$
\begin{aligned}
\mathcal{T} = \frac{G_F}{\sqrt{2}}\Big\{ & V_{ub}V^*_{uq}\Big[a_1(\bar{q}u)_{V-A}\otimes(\bar{u}b)_{V-A} + a_2(\bar{u}u)_{V-A}\otimes(\bar{q}b)_{V-A}\Big] \\
& + V_{cb}V^*_{cq}\Big[a_1(\bar{q}c)_{V-A}\otimes(\bar{c}b)_{V-A} + a_2(\bar{c}c)_{V-A}\otimes(\bar{q}b)_{V-A}\Big] \\
& - V_{tb}V^*_{tq}\Big[a_3\sum_{q'}(\bar{q}'q')_{V-A}\otimes(\bar{q}b)_{V-A} + a_4\sum_{q'}(\bar{q}q')_{V-A}\otimes(\bar{q}'b)_{V-A} \\
& \quad + a_5\sum_{q'}(\bar{q}'q')_{V+A}\otimes(\bar{q}b)_{V-A} - 2a_6\sum_{q'}(\bar{q}q')_{S+P}\otimes(\bar{q}'b)_{S-P} \\
& \quad + \frac{3}{2}a_7\sum_{q'}e_{q'}(\bar{q}'q')_{V+A}\otimes(\bar{q}b)_{V-A} - 3a_8\sum_{q'}e_{q'}(\bar{q}q')_{S+P}\otimes(\bar{q}'b)_{S-P} \\
& \quad + \frac{3}{2}a_9\sum_{q'}e_{q'}(\bar{q}'q')_{V-A}\otimes(\bar{q}b)_{V-A} + \frac{3}{2}a_{10}\sum_{q'}e_{q'}(\bar{q}q')_{V+A}\otimes(\bar{q}'b)_{V-A}\Big]\Big\},
\end{aligned}
\tag{2.6}
$$

where the symbol $\otimes$ stands for $\langle MX|j_1\otimes j_2|B\rangle = \langle M|j_1|0\rangle\langle X|j_2|B\rangle$ or $\langle X|j_1|0\rangle\langle M|j_2|B\rangle$.

The coefficients in Eq. (2.6) evaluated in the naive dimensional regularization (NDR) scheme for γ_5 have the expressions:

$$
\begin{aligned}
a_1 &= c_1 + \frac{c_2}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_2\,F, \\
a_2 &= c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_1\,F, \\
a_3 &= c_3 + \frac{c_4}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_4\,F, \\
a_4 &= c_4 + \frac{c_3}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\Big\{c_3[F + G(s_q) + G(s_b) + \frac{4}{3}] - c_1\left(\frac{\lambda_u}{\lambda_t}G(s_u) + \frac{\lambda_c}{\lambda_t}G(s_c) - \frac{2}{3}\right) \\
&\quad + (c_4+c_6)\sum_{i=u}^{b}G(s_i) + \frac{3}{2}(c_8+c_{10})\sum_{i=u}^{b}e_iG(s_i) - \frac{1}{2}c_9[G(s_q) + G(s_b) + \frac{4}{3}] + c_gG_g\Big\}, \\
a_5 &= c_5 + \frac{c_6}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_6(-F-12), \\
a_6 &= c_6 + \frac{c_5}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\Big\{-c_1\left(\frac{\lambda_u}{\lambda_t}G'(s_u) + \frac{\lambda_c}{\lambda_t}G'(s_c) - \frac{2}{3}\right) + c_3[G'(s_q) + G'(s_b) + \frac{4}{3}] - 6c_5 \\
&\quad + (c_4+c_6)\sum_{i=u}^{b}G'(s_i) + \frac{3}{2}(c_8+c_{10})\sum_{i=u}^{b}e_iG'(s_i) - \frac{1}{2}c_9[G'(s_q) + G'(s_b) + \frac{4}{3}] + c_gG'_g\Big\},
\end{aligned}
\tag{2.7}
$$

$$
\begin{aligned}
a_7 &= c_7 + \frac{c_8}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_8(-F-12) - \frac{\alpha}{9\pi}\,N_cC_e,\\
a_8 &= c_8 + \frac{c_7}{N_c} - 6\,\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_7 - 6\frac{\alpha}{9\pi}\,C'_e,\\
a_9 &= c_9 + \frac{c_{10}}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_{10}\,F - \frac{\alpha}{9\pi}\,N_cC_e,\\
a_{10} &= c_{10} + \frac{c_9}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_9\,F - \frac{\alpha}{9\pi}\,C_e,
\end{aligned}
$$

where $C_F = (N_c^2-1)/(2N_c)$, $s_i = m_i^2/m_b^2$, and $\lambda_q = V_{qb}V^*_{qq'}$.

In Eq. (2.7), the vertex correction in the NDR scheme is given by [1]

$$
F = -12\ln\frac{\mu}{m_b} - 18 + f_I, \tag{2.8}
$$

with

$$
f_I = \int_0^1 dx\,\Phi^M(x)\left(3\frac{1-2x}{1-x}\ln x - 3i\pi\right), \tag{2.9}
$$

where $\Phi^M(x)$ is the leading twist light-cone distribution amplitude (LCDA) of the light meson M. For the vector meson V, $\Phi^V(x)$ is dominated by the longitudinal DA ($\Phi^V_\parallel(x)$) as the contribution from the transverse LCDA ($\Phi^V_\perp(x)$) is suppressed by a factor of m_V/m_b. If the emitted meson is the J/ψ meson, then one has to take into account the charmed quark mass effect [6,7]:

$$
\begin{aligned}
f_I^{J/\psi} = \int_0^1 dx\,\Phi_\parallel^{J/\psi}(x)\Bigg\{&3\frac{1-2x}{1-x}\ln x - 3i\pi + 3\ln(1-z) + 2\frac{z(1-x)}{1-zx}\\
&+\left(\frac{1-x}{(1-zx)^2} - \frac{x}{[1-z(1-x)]^2}\right)z^2x\ln zx + \frac{z^2x^2[\ln(1-z)-i\pi]}{[1-z(1-x)]^2}\\
&+4rz\left[\left(\frac{1}{1-z(1-x)} - \frac{1}{1-zx}\right)\ln zx - \frac{\ln(1-z)-i\pi}{1-z(1-x)}\right]\Bigg\},
\end{aligned} \tag{2.10}
$$

where $z = m^2_{J/\psi}/m^2_B$ and $r = (f^T_{J/\psi}m_c)/(f_{J/\psi}m_{J/\psi})$, with $f^T_{J/\psi}$ being the tensor decay constant of the J/ψ defined by

$$
\langle J/\psi(P,\lambda)|\bar c\sigma_{\mu\nu}c|0\rangle = -if^T_{J/\psi}(\varepsilon^{*(\lambda)}_\mu P_\nu - \varepsilon^{*(\lambda)}_\nu P_\mu). \tag{2.11}
$$

Note that the third line in Eq. (2.10) arises from the transverse polarization component of the J/ψ. Since the asymptotic form of the distribution amplitudes $\Phi_\perp^{J/\psi}(x)$ and $\Phi_\parallel^{J/\psi}(x)$ is the same, namely $6x(1-x)$, we will thus assume $\Phi_\perp^{J/\psi}(x) = \Phi_\parallel^{J/\psi}(x)$ in general. It should be stressed that the strong phase in f_I or $f_I^{J/\psi}$ comes from final-state rescattering due to the hard gluon exchange between the outgoing M and q.

There are QCD penguin-type diagrams induced by the 4-quark operators O_i for $i = 1,3,4,6,8,9,10$. The corrections are described by the penguin-loop functions $G(s)$ and $G'(s) = G_p(s) + G_\sigma(s)$ given by

$$
\begin{aligned}
G(s) &= -\frac{4}{3}\ln\frac{\mu}{m_b} + 4\int_0^1 dx\,\Phi^M(x)\int_0^1 du\,u(1-u)\ln[s - xu(1-u)], \\
G_p(s) &= -\ln\frac{\mu}{m_b} + 3\int_0^1 dx\,\Phi_p^P(x)\int_0^1 du\,u(1-u)\ln[s - xu(1-u)], \\
G_\sigma(s) &= -\frac{1}{3}\ln\frac{\mu}{m_b} + \frac{1}{3}\int_0^1 dx\,\frac{\Phi_\sigma^P(x)}{x}\int_0^1 du\,u(1-u) \\
&\quad\times\Big\{\ln[s - xu(1-u)] - \frac{1}{2}\,\frac{xu(1-u)}{s - xu(1-u)}\Big\}, \qquad (2.12)
\end{aligned}
$$

where Φ_p^P and Φ_σ^P are the twist-3 LCDAs of a pseudoscalar meson P, to which we will come back shortly. It should be stressed that the penguin-loop contribution proportional to $G'(s)$ is available only if the emitted meson is of the pseudoscalar type. In Eq. (2.7) we have also included the leading electroweak penguin-type diagrams induced by the operators O_1 and O_2 [8]:

$$
\begin{aligned}
C_e &= \left(\frac{\lambda_u}{\lambda_t}G(s_u) + \frac{\lambda_c}{\lambda_t}G(s_c) - \frac{2}{3}\right)\left(c_2 + \frac{c_1}{N_c}\right), \\
C_e' &= \left(\frac{\lambda_u}{\lambda_t}G'(s_u) + \frac{\lambda_c}{\lambda_t}G'(s_c) - \frac{2}{3}\right)\left(c_2 + \frac{c_1}{N_c}\right). \qquad (2.13)
\end{aligned}
$$

The dipole operator O_g will give a tree-level contribution proportional to

$$
\begin{aligned}
G_g &= -2\int_0^1 dx\,\frac{\Phi^M(x)}{x}, \\
G_g' &= -\frac{3}{2}\int_0^1 dx\,\Phi_p^M(x) - \frac{1}{6}\int_0^1 dx\,\frac{\Phi_\sigma^M(x)}{x}. \qquad (2.14)
\end{aligned}
$$

It is well known [9] that strong phases can be perturbatively generated from the penguin loop functions $G(x)$ and $G'(x)$. The virtual gluon's momentum squared $k^2 = (1-x)m_B^2$ is no longer an arbitrary parameter; it is convoluted with the emitted meson wave function $\Phi^M(x)$.

The twist-3 DAs ϕ_p^P and ϕ_σ^P are defined in the pseudoscalar and tensor matrix elements [10]:

$$
\begin{aligned}
\langle P(p)|\bar q_1(0) i\gamma_5 q_2(x)|0\rangle &= f_P\mu_\chi^P\int_0^1 d\bar\eta\, e^{i\bar\eta p\cdot x}\phi_p^P(\bar\eta), \\
\langle P(p)|\bar q_1(0)\sigma_{\mu\nu}\gamma_5 q_2(x)|0\rangle &= -\frac{i}{6} f_P\mu_\chi^P\left[1 - \left(\frac{m_1+m_2}{m_P}\right)^2\right] \\
&\quad\times(p_\mu x_\nu - p_\nu x_\mu)\int_0^1 d\bar\eta\, e^{i\bar\eta p\cdot x}\phi_\sigma^P(\bar\eta), \qquad (2.15)
\end{aligned}
$$

where $\mu_\chi^P = m_P^2/(m_1+m_2)$ is proportional to the chiral condensate. In the present paper we will take the asymptotic forms for twist-2 and twist-3 distribution amplitudes:

$$
\Phi^{P,V,J/\psi}(x) = 6x(1-x), \qquad \Phi_p^P(x) = 1, \qquad \Phi_\sigma^P(x) = 6x(1-x). \qquad (2.16)
$$

Several remarks are in order. (i) The coefficients a_{8a} and a_{10a} [11] induced by the electroweak penguin operators $O_{8,9,10}$ are absorbed in our case into a_6 and a_4 in Eq. (2.7). The contributions of C_e and C_e' to the electroweak coefficients a_{7-10}, which have been neglected in most recent literature, are numerically more important than a_{8a} and a_{10a} owing to the large Wilson coefficients c_1 and c_2. (ii) The scale and γ_5-scheme dependence of the Wilson coefficients $c_i(\mu)$ is canceled by the perturbative radiative corrections. In particular it is easily seen that the scale dependence of a_i is compensated by the logarithmic μ dependence in F. However, note that the coefficients a_6 and a_8 are scale dependent. This is because the hadronic matrix element of $(S-P)(S+P)$ operator is proportional to μ_χ^P/m_b which is also scale dependent owing to the running quark masses, the scale dependence of a_6 and a_8 is canceled by the corresponding one in $\mu_\chi(\mu)/m_b(\mu)$. We have included penguin-type corrections to $a_{4,6,8}$. Moreover, we have included the new contributions from the twist-3 DA $\Phi_\sigma(x)$. (iii) In the generalized factorization approach for nonleptonic decays, the nonfactorized effects are sometimes parametrized in terms of the effective number of colors $N_c^{\rm eff}$ so that

$$a_{2i}=c_{2i}+\frac{1}{(N_c^{\rm eff})_{2i}}c_{2i-1}, \qquad a_{2i-1}=c_{2i-1}+\frac{1}{(N_c^{\rm eff})_{2i-1}}c_{2i}. \tag{2.17}$$

Furthermore, it is assumed that $(N_c^{\rm eff})_i$ is process independent. In the improved QCD factorization approach, a_i (see Table II) and hence $(N_c^{\rm eff})_i$ are in general complex and they are process and i dependent.

III. TWO-BODY DECAYS OF THE b QUARK

The general decay amplitudes for $b\to Pq$ and $b\to Vq$ read

$$\begin{aligned}
A(b\to Pq) &= i\frac{G_F}{\sqrt{2}}\Big\{(A^tV_{ub}V_{uq'}^*-A^pV_{tb}V_{tq'}^*)f_Pp_P^\mu\,\bar q\gamma_\mu(1-\gamma_5)b \\
&\qquad -B\,V_{tb}V_{tq'}^*f_P\,\bar q(1-\gamma_5)b\Big\}, \\
A(b\to Pc) &= i\frac{G_F}{\sqrt{2}}\,A^tV_{cb}V_{uq'}^*f_Pp_P^\mu\,\bar c\gamma_\mu(1-\gamma_5)b, \\
A(b\to Vq) &= \frac{G_F}{\sqrt{2}}(\tilde A^tV_{ub}V_{uq'}^*-\tilde A^pV_{tb}V_{tq'}^*)f_Vm_V\varepsilon_V^{*\mu}\,\bar q\gamma_\mu(1-\gamma_5)b, \\
A(b\to Vc) &= \frac{G_F}{\sqrt{2}}\,\tilde A^tV_{cb}V_{uq'}^*f_Vm_V\varepsilon_V^{*\mu}\,\bar q\gamma_\mu(1-\gamma_5)b, \\
A(b\to J/\psi\, q) &= \frac{G_F}{\sqrt{2}}(\tilde A^tV_{cb}V_{cq}^*-\tilde A^pV_{tb}V_{tq}^*)f_{J/\psi}m_{J/\psi}\varepsilon_{J/\psi}^{*\mu}\,\bar q\gamma_\mu(1-\gamma_5)b,
\end{aligned} \tag{3.1}$$

where $q'=d,s$ is not necessarily the same as q, and the superscripts t and p denote tree and penguin contributions, respectively. The coefficients A and B relevant for some two-body

hadronic b decay modes of interest are summarized in Table I. Owing to the complications for the η and η' production, their cases will be discussed separately below. Note that the coefficient B proportional to μ_χ/m_b is formally power suppressed in the heavy quark limit, but numerically it is important since $\mu_\chi/m_b \sim 12\Lambda_{\rm QCD}/m_b$ [see a discussion after Eq. (3.15)]. Therefore, we will keep this calculable power correction.

The decay amplitudes of $b \to \eta^{(\prime)}s,\ \eta^{(\prime)}d$ have the expressions

$$
\begin{aligned}
A(b\to\eta^{(\prime)}s) = i\frac{G_F}{\sqrt{2}}\Bigg\{\Bigg[& V_{ub}V_{us}^*a_2f^u_{\eta^{(\prime)}} + V_{cb}V_{cs}^*a_2f^c_{\eta^{(\prime)}} - V_{tb}V_{ts}^*\Big((a_3-a_5-a_7+a_9)f^c_{\eta^{(\prime)}} \\
& +[a_3+a_4-a_5+\frac{1}{2}(a_7-a_9-a_{10})]f^s_{\eta^{(\prime)}} \\
& +(2a_3-2a_5-\frac{1}{2}a_7+\frac{1}{2}a_9)f^u_{\eta^{(\prime)}}\Big)\Bigg]\, p^\mu_{\eta^{(\prime)}}\bar{s}\gamma_\mu(1-\gamma_5)b \\
& +(2a_6-a_8)\frac{m^2_{\eta^{(\prime)}}}{2m_s(m_b-m_s)}(f^s_{\eta^{(\prime)}}-f^u_{\eta^{(\prime)}})\bar{s}(1-\gamma_5)b\Bigg\},
\end{aligned}
\tag{3.2}
$$

and

$$
\begin{aligned}
A(b\to\eta^{(\prime)}d) = i\frac{G_F}{\sqrt{2}}\Bigg\{\Bigg[& V_{ub}V_{ud}^*a_2f^u_{\eta^{(\prime)}} + V_{cb}V_{cd}^*a_2f^c_{\eta^{(\prime)}} - V_{tb}V_{td}^*\Big((a_3-a_5-a_7+a_9)f^c_{\eta^{(\prime)}} \\
& +[2a_3+a_4-2a_5+\frac{1}{2}(-a_7+a_9-a_{10})]f^u_{\eta^{(\prime)}} \\
& +[a_3-a_5+\frac{1}{2}(a_7-a_9)]f^s_{\eta^{(\prime)}}\Big)\Bigg]\, p^\mu_{\eta^{(\prime)}}\bar{d}\gamma_\mu(1-\gamma_5)b \\
& +(2a_6-a_8)\frac{m^2_{\eta^{(\prime)}}}{2m_s(m_b-m_d)}(f^s_{\eta^{(\prime)}}-f^u_{\eta^{(\prime)}})r_{\eta^{(\prime)}}\bar{d}(1-\gamma_5)b\Bigg\},
\end{aligned}
\tag{3.3}
$$

where the decay constants of the η and η' are defined by $\langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}\rangle = if^q_{\eta^{(\prime)}}p_\mu$. The η' decay constants follow a two-angle mixing pattern [12,13]:

$$
f^u_{\eta'} = \frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \qquad f^s_{\eta'} = -2\frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \tag{3.4}
$$

with f_8 and f_0 being the decay constants of the SU(3) octet and singlet η_8 and η_0, respectively:

$$
\langle 0|A^0_\mu|\eta_0\rangle = if_0p_\mu, \qquad \langle 0|A^8_\mu|\eta_8\rangle = if_8p_\mu. \tag{3.5}
$$

Likewise, for the η meson

$$
f^u_{\eta} = \frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0, \qquad f^s_{\eta} = -2\frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0. \tag{3.6}
$$

It must be emphasized that the two-mixing angle formalism proposed in [12,13] applies to the decay constants of the η' and η rather than to their wave functions. It is found in [13] that phenomenologically

$$\theta_8 = -21.2^\circ, \qquad \theta_0 = -9.2^\circ, \qquad f_8/f_\pi = 1.26, \qquad f_0/f_\pi = 1.17. \tag{3.7}$$

Numerically, we obtain

$$f_\eta^{u,d} = 78\,\mathrm{MeV}, \quad f_\eta^s = -112\,\mathrm{MeV}, \quad f_{\eta'}^{u,d} = 63\,\mathrm{MeV}, \quad f_{\eta'}^s = 137\,\mathrm{MeV}. \tag{3.8}$$

The decay constant $f_{\eta'}^c$, defined by $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'\rangle = i f_{\eta'}^c q_\mu$, is related to the intrinsic charm content of the η' and it has been estimated from theoretical calculations [14] and from the phenomenological analysis of the data on $J/\psi \to \eta_c\gamma$, $J/\psi \to \eta'\gamma$ and of the $\eta\gamma$ and $\eta'\gamma$ transition form factors [8,13,15]; it is expected to lie in the range –2.0 MeV $\leq f_{\eta'}^c \leq$ –18.4 MeV. In this paper we use the values

$$f_{\eta'}^c = -(6.3 \pm 0.6)\,\mathrm{MeV}, \qquad f_\eta^c = -(2.4 \pm 0.2)\,\mathrm{MeV}, \tag{3.9}$$

as obtained from a phenomenological analysis performed in [13].

Care must be taken to consider the pseudoscalar matrix element for $\eta^{(\prime)} \to$ vacuum transition. The anomaly effects must be included in order to ensure a correct chiral behavior for the pseudoscalar matrix element [16]. The results are [17,8]

$$\begin{aligned}
\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle &= -i\frac{m^2_{\eta^{(\prime)}}}{2m_s}\left(f^s_{\eta^{(\prime)}} - f^u_{\eta^{(\prime)}}\right), \\
\langle \eta^{(\prime)}|\bar{u}\gamma_5 u|0\rangle &= \langle \eta^{(\prime)}|\bar{d}\gamma_5 d|0\rangle = r_{\eta^{(\prime)}}\,\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle,
\end{aligned} \tag{3.10}$$

with [16]

$$\begin{aligned}
r_{\eta'} &= \frac{\sqrt{2f_0^2 - f_8^2}}{\sqrt{2f_8^2 - f_0^2}}\,\frac{\cos\theta + \frac{1}{\sqrt{2}}\sin\theta}{\cos\theta - \sqrt{2}\sin\theta}, \\
r_\eta &= -\frac{1}{2}\,\frac{\sqrt{2f_0^2 - f_8^2}}{\sqrt{2f_8^2 - f_0^2}}\,\frac{\cos\theta - \sqrt{2}\sin\theta}{\cos\theta + \frac{1}{\sqrt{2}}\sin\theta},
\end{aligned} \tag{3.11}$$

where θ is the $\eta - \eta'$ mixing angle:

$$\eta' = \eta_8\sin\theta + \eta_0\cos\theta, \qquad \eta = \eta_8\cos\theta - \eta_0\sin\theta. \tag{3.12}$$

We will use $\theta = -15.4^\circ$ as determined in [13].

To proceed with numerical calculations, we need to specify some input parameters. For Wilson coefficients, we use the next-to-leading ones in the NDR scheme

$$\begin{aligned}
&c_1 = 1.082, \quad c_2 = -0.185, \quad c_3 = 0.014, \quad c_4 = -0.035, \quad c_5 = 0.009, \quad c_6 = -0.041, \\
&c_7/\alpha = -0.002, \quad c_8/\alpha = 0.054, \quad c_9/\alpha = -1.292, \quad c_{10}/\alpha = 0.263, \quad c_g = -0.143,
\end{aligned} \tag{3.13}$$

with α being the electromagnetic fine-structure coupling constant. These values taken from Table XXII of [18] are evaluated at $\mu = \overline{m}_b(m_b) = 4.40$ GeV and $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 225$ MeV. For the decay constants other than η and η' we employ

$$f_\pi = 132\,\mathrm{MeV}, \qquad f_K = 160\,\mathrm{MeV}, \qquad f_\rho = 216\,\mathrm{MeV}, \qquad f_{K^*} = 221\,\mathrm{MeV},$$
$$f_\omega = 195\,\mathrm{MeV}, \qquad f_\phi = 237\,\mathrm{MeV}, \qquad f_{J/\psi} = 405\,\mathrm{MeV}. \tag{3.14}$$

For the CKM matrix elements, we take $|V_{cb}| = 0.0395$ and $|V_{ub}/V_{cb}| = 0.085$. For the unitarity angle γ we use $\gamma = 60°$ as extracted from recent global CKM fits [19]. The other two unitarity angles α and β are then fixed.

The hadronic matrix element of the $(S-P)(S+P)$ operator involves the quantity $\mu_\chi^P/\overline{m}_b$

$$\frac{\mu_\chi^P}{\overline{m}_b(\mu)} = \frac{m_P^2}{\overline{m}_b(\mu)[\overline{m}_1(\mu) + \overline{m}_2(\mu)]}, \tag{3.15}$$

where $\overline{m}_q$ is the running quark mass of the quark q, and hence it is formally $\Lambda_{\rm QCD}/\overline{m}_b$ power suppressed in the heavy quark limit. However, numerically it is chirally enhanced. At $\mu = m_b$ scale, we have (see [20] for the running quark mass ratios and evolution)

$$\overline{m}_s(m_b) = 90\,\mathrm{MeV}, \qquad \overline{m}_d(m_b) = 4.6\,\mathrm{MeV}, \qquad \overline{m}_u(m_b) = 2.3\,\mathrm{MeV}, \tag{3.16}$$

corresponding to $\overline{m}_s(1\,\mathrm{GeV}) = 140\,\mathrm{MeV}$ or $\overline{m}_s(2\,\mathrm{GeV}) = 101\,\mathrm{MeV}$. We will neglect the small flavor-SU(3) breaking for the chiral condensate and use the averaged value $\mu_\chi(m_b) = 2.7$ GeV.* As a consequence, $\mu_\chi/\overline{m}_b = 12\Lambda_{\rm QCD}/\overline{m}_b$ yields a large chiral enhancement. Apart from the current quark masses appearing in the use of equations of motion, we will utilize the pole masses for the heavy quarks: $m_b = 4.8$ GeV and $m_c = 1.5$ GeV. To compute the branching ratios, we use the mean lifetime of the admixture of bottom particles: $\tau = 1.564 \times 10^{-12}s$ [21].

Since the coefficients $a_{4,6-10}$ involve the CKM matrix elements $\lambda_{u,c}/\lambda_t$ [see Eq. (2.7)], the results of the coefficients a_i are exhibited in Table II for $b \to P\,q$ and $q' = d, s$. It is evident that $a_{3,4,5,6}$ are enhanced substantially compared to the naive factorization values and have large imaginary parts and that a_2 in QCD factorization becomes very small. In particular, the calculated coefficient $a_2(Pq) = 0.02 - 0.08i$ in QCD factorization is dramatically different from the value 0.18 obtained in naive factorization. The smallness of a_2 imposes a serious problem. For example, the predicted branching ratio of $b \to J/\psi s$ is too small compared to the experimental value $\mathcal{B}(B \to J/\psi X_s) = (8.0 \pm 0.8) \times 10^{-3}$ [22]. We will come to this point in Sec. IV. Note that a_9 is the dominant electroweak penguin coefficient. Owing to the large cancellation between a_3 and a_5, the decay $b \to \phi\,d$ is electroweak penguin dominated. Likewise, $b \to \pi^0 s$ and $\rho^0 s$ are also dominated by electroweak penguin diagrams.

---

*A value as small as $\mu_\chi = 1.4$ GeV is sometimes chosen in the literature. However, we favor the higher value given above as one can then readily account for the observed $B \to K\pi$ rates, which are difficult to explain in terms of the smaller μ_χ.

We are now ready to compute the decay rates using,

$$
\begin{aligned}
\Gamma(b \to Pq) &= \frac{G_F^2 f_P^2 m_b^3}{16\pi}\Big(|A^t V_{ub}V_{uq'}^* - A^p V_{tb}V_{tq'}^*|^2 + |B\, V_{tb}V_{tq'}^*|^2\Big)\left(1 - \frac{m_P^2}{m_b^2}\right), \\
\Gamma(b \to Pc) &= \frac{G_F^2 f_P^2 m_b^3}{16\pi}\,|A^t V_{cb}V_{uq'}^*|^2 \left(1 + \frac{m_c^2}{m_b^2} - \frac{m_P^2}{m_b^2}\right), \\
\Gamma(b \to Vq) &= \frac{G_F^2 f_V^2 m_b^3}{16\pi}|\tilde{A}^t V_{ub}V_{uq'}^* - \tilde{A}^p V_{tb}V_{tq'}^*|^2 \left(1 + \frac{m_V^2}{m_b^2} - 2\frac{m_V^4}{m_b^4}\right), \\
\Gamma(b \to Vc) &= \frac{G_F^2 f_V^2 m_b^3}{16\pi}|\tilde{A}^t V_{cb}V_{uq'}^*|^2 \left(1 - \frac{m_c^2 - m_V^2}{m_b^2} + \frac{m_c^4 - 2m_V^4}{m_b^4}\right), \\
\Gamma(b \to J/\psi\, q) &= \frac{G_F^2 f_{J/\psi}^2 m_b^3}{16\pi}|\tilde{A}^t V_{cb}V_{cq}^* - \tilde{A}^p V_{tb}V_{tq}^*|^2 \left(1 + \frac{m_{J/\psi}^2}{m_b^2} - 2\frac{m_{J/\psi}^4}{m_b^4}\right).
\end{aligned}
\tag{3.17}
$$

The expression of $\Gamma(b \to \eta^{(\prime)}q)$ is similar to that of $\Gamma(b \to Pq)$. We will also study the CP-violating partial-rate asymmetry (PRA) defined by

$$
\mathcal{A} = \frac{\Gamma(b \to Mq) - \Gamma(\bar{b} \to \bar{M}\bar{q})}{\Gamma(b \to Mq) + \Gamma(\bar{b} \to \bar{M}\bar{q})}. \tag{3.18}
$$

The CP-averaged branching ratios and direct CP-violating partial rate asymmetries for some two-body hadronic b decays of interest are summarized in Table III. Compared to the predictions of branching ratios based on naive factorization [4], there are three major modifications: (i) Decay modes $\pi^- u$, $\bar{K}^0 d$, $\bar{K}^{*0} d$ and $K^- u$ are significantly enhanced owing to the large penguin coefficients a_6 and a_4. (ii) The modes $\pi^0 d$, $\rho^0 d$, $\omega\, d$, $J/\psi s$, $J/\psi d$ with neutral emitted mesons are suppressed relative to the naive factorization ones due to the smallness of a_2. (iii) The ϕd mode has a smaller rate due to the large cancellation between a_3 and a_5. That is, while ϕd is QCD-penguin dominated in naive factorization, it becomes electroweak-penguin dominated in QCD factorization.

For the prompt η' production in semi-inclusive decays, we find the four-quark operator contributions to $b \to \eta' s$ can only account for about 10% of the measured result [23]:

$$
\mathcal{B}(B \to \eta' X_s) = (6.2 \pm 1.6 \pm 1.3^{+0.0}_{-1.5}(\mathrm{bkg})) \times 10^{-4} \qquad \text{for } 2.0 < p_{\eta'} < 2.7\ \mathrm{GeV/c}, \tag{3.19}
$$

where X_s is the final state containing a strange quark. One important reason is that there is an anomaly effect in the matrix element $\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle$ manifested by the decay constant $f_{\eta'}^u$ [see Eq. (3.10)]. Since $f_{\eta'}^u \sim \frac{1}{2} f_{\eta'}^s$ [cf. Eq. (3.8)], it is obvious that the decay rate of $b \to \eta' s$ induced by the $(S-P)(S+P)$ penguin interaction is suppressed by the QCD anomaly effect. If there were no QCD anomaly, one would have $\mathcal{B}(b \to \eta' s) = 2.2 \times 10^{-4}$ from four-quark operator contributions which are about one third of the experimental value.

It is useful to explicitly take into account the constraints from the CPT theorem when computing PRA's for inclusive decays at the quark level [24] (for a review, see [25]). Take

$b \to d u\bar{u}$ as an example and note that the penguin amplitude, say $\lambda_t a_4$, can be written as $-(\lambda_u a_4^u + \lambda_c a_4^c)$, where $\lambda_q = V_{qb}V_{qq'}^*$, $a_4^u = -c_1 G(s_u) + \cdots$ and $a_4^c = -c_1 G(s_c) + \cdots$ with the ellipses being the common terms given in Eq. (2.7) for a_4 and $G(s_q)$ the penguin function with the internal quark q [see Eq. (2.12)]. In general, a_4 has absorptive contributions from all u, d, s, c quark loops. It is clear that PRA's arise from the interference between the tree amplitude $\lambda_u a_1$ and the penguin amplitude $\lambda_c a_4^c$ and the one between $\lambda_u a_4^u$ and $\lambda_c a_4^c$. The CPT theorem implies that the "diagonal" strong penguin phases coming from the diagonal process $q + \bar{q} \to q + \bar{q}$ will not contribute to the rate asymmetry [26]. For example, at order α_s the interference between $\lambda_u a_1$ and $\lambda_c a_4^c$ with the absorptive cut from the u quark loop in the penguin diagram does not contribute to PRA in $b \to s(d)u\bar{u}$ decays. It is easily seen that the compensating process for this interference is itself. Likewise, the PRA at order α_s^2 due to the interference between two different penguin amplitudes with the $u\bar{u}$ absorptive cut in the penguin loop will be compensated by the one between the tree amplitude and the higher order penguin diagram that contains an absorptive part from the $u\bar{u}$ quark loop inset in the gluon propagator. Therefore, one has to disregard the penguin phases coming from $G(s_u)$ and $G'(s_u)$ for the PRA's in the decay $b \to d u\bar{u}$. Similarly, the phase of the s-loop penguin diagram should be dropped in rate-asymmetry calculations of $b \to d s\bar{s}$ and $b \to s s\bar{s}$ in order to be consistent with the requirement of the CPT theorem. By the same token, the strong "diagonal" phase in coefficients a_i due to final-state hard gluon exchange [see Eq. (2.9)] will not contribute to rate asymmetries in quark level processes.

The implication of the CPT theorem for PRA's at the hadron level in exclusive or semi-inclusive reactions is more complicated [27]. Consider the above example $b \to d u\bar{u}$ again. The corresponding semi-inclusive decays of the b quark can be manifested as $b \to (\pi^-, \rho^-)u$ and $b \to (\pi^0, \rho^0, \omega)d$ at the two-body level and $(\pi^-\pi^0, K^0K^-)u$, $(\pi^+\pi^-, \pi^0\pi^0, K^+K^-)d$ at the three-body level and etc. The CPT theorem no longer constrains the absorptive cut from the u-loop penguin diagram not to contribute separately to each aforementioned semi-inclusive b decay, though the cancellation between $u\bar{u}$ and $c\bar{c}$ quarks will occur when all semi-inclusive modes are summed over. In view of this observation, we shall keep all the strong phases in the calculation of direct CP violation in the individual semi-inclusive decay.

The presence of the strong phase in the hard kernel f_I [Eq. (2.9)] in QCD factorization has several noticeable effects: (i) The rate asymmetries in the decays $b \to \phi d$, $(\pi^0, \rho^0, \omega)s$ vanish in naive factorization because the coefficients a_2 and $a_{3,5,7,9,10}$ there are real. In QCD factorization, the large phase of a_2 (see Table II) will yield large PRA's for these modes. (ii) The color-suppressed tree-dominated decays $b \to (\pi^0, \rho^0, \omega)d$ have large PRA's due to the large imaginary part of a_2 and the smallness of $|a_2|$. To see this, we consider $b \to \rho^0 d$ as an illustration. The partial rate difference $\Delta\Gamma(b \to \rho^0 d) = \Gamma(b \to \rho^0 d) - \Gamma(\bar{b} \to \rho^0 \bar{d})$ is proportional to (see Table I for the amplitude)

$$\Delta\Gamma(b\to\rho^0 d)\propto \mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td})\,\mathrm{Im}\left[a_2(-a_4+\frac{3}{2}a_7+\frac{3}{2}a_9+\frac{1}{2}a_{10})\right]. \tag{3.20}$$

Since a_2 is dominated by the imaginary part, it follows that

$$\Delta\Gamma(b\to\rho^0 d)\propto \sin\alpha\,\mathrm{Im}a_2\,\mathrm{Re}\left(-a_4+\frac{3}{2}a_7+\frac{3}{2}a_9+\frac{1}{2}a_{10}\right). \tag{3.21}$$

Because $|V_{ub}V_{ud}^*|\gg|V_{tb}V_{td}^*|$ and $|a_2|$ is small, it is clear that $b\to\rho^0 d$ has a large PRA governed by the vertex-induced strong phase. In contrast, the rate asymmetry in penguin-dominated modes is largely due to the strong penguin phase. Consider the process $b\to K^{*-}u$. The partial rate difference is

$$\begin{aligned}\Delta\Gamma(b\to K^{*-}u) &\propto \mathrm{Im}(V_{ub}V_{us}^*V_{cb}^*V_{cs})\mathrm{Im}[a_1a_4^c+a_4^ua_4^c] \qquad (3.22)\\ &\approx \sin\gamma\,[\mathrm{Im}a_1\,\mathrm{Re}a_4^c-c_1\mathrm{Im}G(s_c)\mathrm{Re}a_1-c_1\mathrm{Im}G(s_u)\mathrm{Re}a_4^c-c_1\mathrm{Im}G(s_c)\mathrm{Re}a_4^u],\end{aligned}$$

where we have applied the relation $\lambda_t a_4=-(\lambda_u a_4^u+\lambda_c a_4^c)$ as given before. Since the imaginary part of a_1 is very small (see Table II), it is evident that CP asymmetries in the penguin-dominated modes are governed by the strong penguin phase.

In the present QCD factorization approach we have considered the penguin corrections to $a_{4,6-10}$ induced by not only tree operators but also QCD and electroweak penguin operators. For example, the parameters $a_{7,9,10}$ do receive an absorptive contribution via electroweak penguin-type diagrams generated by tree 4-quark operators $O_{1,2}$ [see Eq. (2.7)]. Therefore, they receive more penguin phases. Another important difference between QCDF and naive factorization is that the gluon's virtuality k^2 is no longer arbitrary; this tends to remove considerable uncertainties in the estimates of the CP asymmetries. These may account for the general differences between the present results and [4]. Note that our predictions for PRA's in the decays $\bar B^0\to K^-(K^{*-})X$ and $B^-\to \bar K^0(\bar K^{*0})X$ are in agreement with [2] for $\gamma=60^\circ$. (The definition of the rate asymmetry in [2] has a sign opposite to ours.)

IV. SEMI-INCLUSIVE B DECAYS

A major advantage of studying the quasi-two-body decay of the b quark is that it does not involve the unknown form factors and hence the theoretical uncertainty is considerably reduced. In order to retain this merit for semi-inclusive B decays, it is necessary to circumvent the second matrix element term appearing in Eq. (2.3). Fortunately, this can be achieved by choosing a B meson whose spectator quark content is not contained in the outgoing meson M. For example, the counterpart of $b\to\pi^0 d$ at the meson level will be $\overline{B}_s^0\to\pi^0 X$ as then the $\overline{B}_s-\pi^0$ transition does not contribute. In contrast, the decay $B^-\to\pi^0 X$ or $\overline{B}^0\to\pi^0 X$ will involve the unwanted $B-\pi$ form factors. As stressed in Sec.II, it is necessary to impose a cut, say $E_M>2.1$ GeV for the light emitted meson, in

order to reduce contamination from the unwanted background and enhance the presence of the two-body quark decay $b \to Mq$. Therefore, in the absence of the bound state effect it is expected that, for example, $\Gamma(\bar{B}^0 \to \pi^- X) \approx \Gamma(b \to \pi^- u)$ after applying the parton-model approximation

$$\sum_X |X\rangle\langle X| \approx \sum_s \int \frac{d^3p}{(2\pi)^3 2E_u} |u(p_u, s)\rangle\langle u(p_u, s)|. \tag{4.1}$$

There are two more complications for semi-inclusive B decays. First, $B \to MX$ can be viewed as the two-body decay $b \to Mq$ in the heavy quark limit. For the finite b quark mass, it becomes necessary to consider the initial b quark bound state effect. Second, consider the 3-body decay $\overline{B} \to Mq_1\bar{q}_2$ with the quark content $(b\bar{q}_2)$ for the $\bar{B}$ meson. One needs a hard gluon exchange between the spectator quark $\bar{q}_2$ and the meson M in order to ensure that the outgoing $\bar{q}_2$ is hard. For exclusive two-body decays, the nonfactorizable hard spectator contribution is customarily denoted as [1]

$$\frac{G_F}{\sqrt{2}} \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_i f_{II}. \tag{4.2}$$

Numerically, f_{II} is much larger than f_I for exclusive decays. For the semi-inclusive case at hand, it has been argued that f_{II} is subject to a phase-space suppression since it involves three particles in the final states rather than the two-body one for f_I [3]. However, we shall see below that it is not the case for color-suppressed decay modes.

A. Initial bound state effect

The initial bound state effects on branching ratios and CP asymmetries have been studied recently in [2] using two different approaches: the light-cone expansion approach and the heavy quark effective theory approach. We will follow [2] to employ the second approach which amounts to modifying the decay rates by

$$\begin{aligned}
\Gamma(B \to PX) &= \frac{G_F f_P^2 m_b^3}{16\pi}\Big\{|A^t V_{ub} V_{uq'}^* - A^p V_{tb} V_{tq'}^*|^2 \eta_1 + |B\, V_{tb} V_{tq'}^*|^2 \eta_2\Big\}\left(1 - \frac{m_P^2}{m_B^2}\right), \\
\Gamma(B \to VX) &= \frac{G_F f_V^2 m_b^3}{16\pi}|\tilde{A}^t V_{ub} V_{uq'}^* - \tilde{A}^p V_{tb} V_{tq'}^*|^2 \eta_1 \left(1 + \frac{m_V^2}{m_B^2} - 2\frac{m_V^4}{m_B^4}\right), \\
\Gamma(B \to J/\psi\, X) &= \frac{G_F f_{J/\psi}^2 m_b^3}{16\pi}|\tilde{A}^t V_{cb} V_{cq}^* - \tilde{A}^p V_{tb} V_{tq}^*|^2 \eta_1 \left(1 + \frac{m_{J/\psi}^2}{m_B^2} - 2\frac{m_{J/\psi}^4}{m_B^4}\right),
\end{aligned} \tag{4.3}$$

where

$$\eta_1 = \left(1 + \frac{7}{6}\frac{\mu_G^2}{m_b^2} - \frac{53}{6}\frac{\mu_\pi^2}{m_b^2}\right), \qquad \eta_2 = \left(1 - \frac{\mu_G^2}{2m_b^2} + \frac{\mu_\pi^2}{2m_b^2}\right), \tag{4.4}$$

and

$$\mu_G^2 = \frac{\langle B|\bar{h}G_{\mu\nu}\sigma^{\mu\nu}h|B\rangle}{4m_B}, \qquad \mu_\pi^2 = -\frac{\langle B|\bar{h}(iD_\perp)^2h|B\rangle}{2m_B}, \tag{4.5}$$

with $D_\perp^\mu = D^\mu - v^\mu v\cdot D$, where v is the four-velocity of the B meson. The nonperturbative HQET parameter μ_G^2 is fixed from the B^*-B mass splitting to be $0.36\,\mathrm{GeV}^2$. Following [2] we use $\mu_\pi^2 = 0.5\,\mathrm{GeV}^2$, which is consistent with QCD sum rule and lattice QCD calculations [28]. Compared to the two-body decays $b\to Mq$ shown in Table III, we see that the branching ratio of $B\to PX$ and $B\to VX$ owing to bound state effects is reduced by a factor of $(5\sim 10)\%$ and 17%, respectively, while the CP asymmetry remains intact for VX decays and for most of PX modes.

B. Nonfactorizable hard spectator interactions

We now turn to the hard spectator interactions in the 3-body decay $B(p_B)\to M(p_M)+q_1(p_1)+\bar{q}_2(p_2)$ with a hard gluon exchange between the spectator quark $\bar{q}_2$ and the meson M. A straightforward calculation yields

$$\begin{aligned}
A_{\rm spect}(B\to Pq_1\bar{q}_2) =& \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\frac{4\pi^2}{N_c}f_Pf_B(A_{sp}^tV_{ub}V_{uq'}^* - A_{sp}^pV_{tb}V_{tq'}^*) \\
&\times\int_0^1 d\xi\,\frac{\Phi^P(\xi)}{\xi}\int_0^1 d\bar\rho\,\frac{\Phi_1^B(\bar\rho)}{\bar\rho}\left(\frac{p_P^\mu\bar{q}_1\gamma_\mu(1-\gamma_5)q_2}{p_P\cdot p_2} - \frac{m_B\bar{q}_1(1+\gamma_5)q_2}{p_B\cdot p_2}\right), \\
A_{\rm spect}(B\to Vq_1\bar{q}_2) =& -i\frac{G_F}{\sqrt{2}}\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\frac{4\pi^2}{N_c}m_Vf_Vf_B(\tilde{A}_{sp}^tV_{ub}V_{uq'}^* - \tilde{A}_{sp}^pV_{tb}V_{tq'}^*) \\
&\times\int_0^1 d\xi\int_0^1 d\bar\rho\,\frac{\Phi^V(\xi)\Phi_1^B(\bar\rho)}{\xi\,\bar\rho\,(p_B\cdot p_2)(p_V\cdot p_2+\xi m_V^2/2)} \\
&\times\Big\{(p_B\cdot p_2-\xi p_B\cdot p_V)\varepsilon^{*\mu}\bar{q}_1\gamma_\mu(1-\gamma_5)q_2 - \xi(\varepsilon^*\cdot p_B)\bar{q}_1\not{p}_V(1-\gamma_5)q_2 \\
&-m_B(\varepsilon^*\cdot p_2)\bar{q}_1(1+\gamma_5)q_2 - \xi m_B\bar{q}_1\not{p}_V\not{\varepsilon}^*(1+\gamma_5)q_2\Big\}, \\
A_{\rm spect}(B\to J/\psi q_1\bar{q}_2) =& -i\frac{G_F}{\sqrt{2}}\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\frac{4\pi^2}{N_c}m_{J/\psi}f_{J/\psi}f_B(\tilde{A}_{sp}^tV_{cb}V_{cq'}^* - \tilde{A}_{sp}^pV_{tb}V_{tq'}^*)\int_0^1 d\xi\int_0^1 d\bar\rho \\
&\times\frac{\Phi^{J/\psi}(\xi)\Phi_1^B(\bar\rho)}{\xi\,\bar\rho\,(p_B\cdot p_2)(p_{J/\psi}\cdot p_2)}\Big\{[p_B\cdot p_2-(\xi-r)p_B\cdot p_{J/\psi}]\varepsilon^{*\mu}\bar{q}_1\gamma_\mu(1-\gamma_5)q_2 \\
&-(\xi-r)(\varepsilon^*\cdot p_B)\bar{q}_1\not{p}_{J/\psi}(1-\gamma_5)q_2 - m_B(\varepsilon^*\cdot p_2)\bar{q}_1(1+\gamma_5)q_2 \\
&-(\xi-r)m_B\bar{q}_1\not{p}_{J/\psi}\not{\varepsilon}^*(1+\gamma_5)q_2\Big\},
\end{aligned} \tag{4.6}$$

where $r = (m_cf_{J/\psi}^T)/(m_{J/\psi}f_{J/\psi})$. In deriving the above equation, we have applied the on-shell conditions $\bar{q}_1\not{p}_1 = 0$, $\not{p}_2q_2 = 0$, the approximation $\bar\rho\approx 0$ [see the discussion after Eq. (4.11)] and the B meson wave function [1]:

$$\langle 0|\bar{q}_\alpha(x) b_\beta(0)|\bar{B}(p)\rangle|_{x_+=x_\perp=0} = -\frac{if_B}{4}[(\not{p}+m_B)\gamma_5]_{\beta\gamma}\int_0^1 d\bar{\rho}\, e^{-i\bar{\rho}p_+x_-}[\Phi_1^B(\bar{\rho})+\not{n}_-\Phi_2^B(\bar{\rho})]_{\gamma\alpha}, \quad (4.7)$$

with $n_- = (1,0,0,-1)$. The expressions for $A_{sp}^{t,p}$ and $\tilde{A}_{sp}^{t,p}$ can be obtained from that of $A^{t,p}$ and $\tilde{A}^{t,p}$ (see Table I) respectively by replacing the coefficient a_{2i} (a_{2i-1}) by the Wilson coefficient c_{2i-1} (c_{2i}). In passing, we note that, in contrast to [5], the 2-body decay $b \to Mq_1$ and the 3-body decay $B \to Mq_1\bar{q}_2$ do not interfere with each other to give contributions to $B \to MX$.

Eq. (4.6) can be applied to two-body exclusive decays. Consider $B \to K\pi$ as an example and this amounts to having $P = K$ and a pion from $q_1\bar{q}_2$. Hence $p_2 = \bar{\eta}p_\pi$, where $\bar{\eta}$ is the momentum fraction of the antiquark in the pion. It follows from (4.6) and (2.15) that

$$A_{\rm spect}(B \to K\pi) \propto \int_0^1 \frac{d\bar{\rho}}{\bar{\rho}}\,\Phi_1^B(\bar{\rho})\int_0^1 \frac{d\xi}{\xi}\,\Phi^K(\xi)\int_0^1 \frac{d\bar{\eta}}{\bar{\eta}}\left[\Phi^\pi(\bar{\eta})+\frac{2\mu_\chi}{m_b}\phi_p^\pi(\bar{\eta})\right], \quad (4.8)$$

which was first obtained in [29]. It is evident that the terms proportional to m_B in Eq. (4.6) give twist-3 contributions. Since the twist-3 distribution amplitude $\Phi_p^\pi(\bar{\eta}) \approx 1$, it does not vanish at the endpoints. Consequently, there is a logarithmic divergence of the $\bar{\eta}$ integral which implies that the spectator interaction in $B \to K\pi$ decay is dominated by soft gluon exchange between the spectator quark and quarks that form the emitted kaon, indicating that QCD factorization breaks down at twist-3 order. The above-mentioned infrared divergent problem does not occur in the semi-inclusive decay, however.

The decay distribution due to hard spectator interactions is given by

$$d\Gamma_{\rm spect} = \frac{1}{(2\pi)^3}\,\frac{1}{32m_B^3}\int |A_{\rm spect}|^2 dm_{12}^2 dm_{23}^2, \quad (4.9)$$

or

$$\frac{d\Gamma_{\rm spect}}{dE_M} = \frac{1}{(2\pi)^3}\,\frac{1}{16m_B^2}\int |A_{\rm spect}|^2 dm_{23}^2, \quad (4.10)$$

where E_M is the energy of the outgoing meson M, and $m_{ij}^2 = (p_i+p_j)^2$ with $p_3 = p_M$. For a given E_M, the range of m_{23}^2 is fixed by kinematics. In order to enhance the possibility that $B \to MX$ originates from a quasi-two-body decay, we impose the energy cutoff $E_M > 2.1$ GeV for light mesons and $E_M > 3.3$ GeV for the J/ψ.

C. Results and discussions

To proceed we apply the initial bound state effect to hard spectator interactions and use the B meson wave function

$$\Phi_1^B(\bar{\rho}) = N_B\bar{\rho}^2(1-\bar{\rho})^2 \exp\left[-\frac{1}{2}\left(\frac{\bar{\rho}m_B}{\omega_B}\right)^2\right], \quad (4.11)$$

with $\omega_B = 0.25$ GeV and N_B being a normalization constant. This B meson wave function corresponds to $\lambda_B = 303$ MeV defined by $\int_0^1 d\bar{\rho}\, \Phi^B(\bar{\rho})/\bar{\rho} \equiv m_B/\lambda_B$ [1]. This can be understood since the B meson wave function is peaked at small $\bar{\rho}$: It is of order $m_B/\Lambda_{\rm QCD}$ at $\bar{\rho} \sim \Lambda_{\rm QCD}/m_B$. Hence, the integral over $\phi_B(\bar{\rho})/\bar{\rho}$ produces an $m_B/\Lambda_{\rm QCD}$ term. As for the parameter r defined after Eq. (2.10), it is equal to 1/2 in the heavy quark limit assuming $f^T_{J/\psi} = f_{J/\psi}$. The results of calculations are shown in Table IV. We see that the tree-dominated color-suppressed modes $(\pi^0, \rho^0, \omega)X_{\bar{s}}$, ϕX, $J/\psi X_s, J/\psi X$ and the penguin-dominated mode $\omega X_{s\bar{s}}$ are dominated by the hard spectator corrections. In particular, the prediction $\mathcal{B}(B \to J/\psi X_s) = 9.6 \times 10^{-3}$ is in agreement with the measurement of a direct inclusive J/ψ production: $(8.0 \pm 0.8) \times 10^{-3}$ by CLEO [22] and $(7.89 \pm 0.10 \pm 0.34) \times 10^{-3}$ by BaBar [30]. This is because the relevant spectator interaction is color allowed, whereas the two-body semi-inclusive decays for these modes are color-suppressed. As a consequence, nonfactorizable hard spectator interactions amount to giving a_2 a large enhancement.

It is instructive to compare the enhancement of the J/ψ production in exclusive and semi-inclusive decays. The hard spectator diagrams denoted by f_{II} have been included in the leading-twist order calculations and it is found that $a_2(J/\psi K) \sim 0.06 - 0.05i$ [6,7]. Therefore, the real part of $a_2(J/\psi K)$ is enhanced by f_{II}, which is numerically much larger than f_I, but it is still too small compared to the experimental value 0.26 ± 0.02 [31]. It has been shown recently that to the twist-3 level, the coefficient $a_2(J/\psi K)$ is largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDA ϕ^K_σ, which are formally power-suppressed but chirally, logarithmically and kinematically enhanced [6]. The major theoretical uncertainty is that the infrared divergent contributions there should be treated in a phenomenological way. In this work we found that it is the same spectator mechanism responsible for the enhancement observed in semi-inclusive decay $B \to J/\psi X_s$, and yet we do not encounter the same infrared problem as occurred in the exclusive case, and terms proportional to m_B in Eq. (4.6) are not power suppressed, rendering the present prediction more reliable and trustworthy. It is conceivable that infrared divergences residing in exclusive decays will be washed out when all possible exclusive modes are summed over.

It is also interesting to notice that after including the spectator corrections, the branching ratios and PRA's for the color-suppressed modes $\overline{B}{}^0_s \to (\pi^0, \rho^0, \omega)X_{\bar{s}}$, $B^- \to \phi X$ are numerically close to that predicted in [4] based on naive factorization (see Table III). Note that the large CP asymmetries in $b \to (\pi^0, \rho^0, \omega)d$ decays (see Table III) are washed out to a large extent at hadron level by spectator interactions. By contrast, the nonfactorizable spectator interaction is in general negligible for penguin dominated (except for $\omega X_{s\bar{s}}$) or color-allowed tree dominated decay modes. The channels $(B^-, \overline{B}{}^0) \to (\pi^0, \rho^0, \omega, \phi)X$ are not listed in Table IV as they involve the unwanted form factors. For example, $B^- \to \pi^0 X$ contains a term $a_2 F^{B\pi}$ and $\overline{B}{}^0 \to \pi^0 X$ has a contribution like $a_4 F^{B\pi}$. Hence, the prediction of $(B^-, \overline{B}{}^0) \to \pi^0 X$ is not as clean as $\overline{B}{}^0_s \to \pi^0 X_{\bar{s}}$. Nevertheless, the former is also dominated

by spectator interactions and is expected to have the same order of magnitude for branching ratios as the latter.

Owing to the presence of $B-\eta(\eta')$ form factors, the decays $B\to(\eta,\eta')X$ are also not listed in Table IV. However, we find that the hard spectator corrections to the prompt η' production in semi-inclusive decays are very small and hence the four-quark operator contributions to $b\to\eta' s$ can only account for about 10% of the measured result, Eq. (3.19). Evidently this implies that one needs a new mechanism (but not necessarily new physics) specific to the η'. It has been advocated that the anomalous coupling of two gluons and η' in the transitions $b\to sg^*$ followed by $g^*\to\eta' g$ and $b\to sg^*g^*$ followed by $g^*g^*\to\eta'$ may explain the excess of the η' production [32,17]. An issue in this study is about the form-factor suppression in the $\eta'-g^*-g^*$ vertex and this has been studied recently in the perturbative QCD hard scattering approach [33]. At the exclusive level, it is well known that the decays $B^\pm\to\eta' K^\pm$ and $\overline{B}^0\to\eta'\overline{K}^0$ have abnormally large branching ratios [21]. In spite of many theoretical uncertainties, it is safe to say that the four-quark operator contribution accounts for at most half of the experimental value and the new mechanism responsible for the prolific η' production in semi-inclusive decay could also play an essential role in $B\to\eta' K$ decay.

From Table IV it is clear that the semi-inclusive decay modes: $\overline{B}^0_s\to(\pi^0,\rho^0,\omega)X_{\bar{s}}$, $\rho^0X_{s\bar{s}}$, $\overline{B}^0\to(K^-X,K^{*-}X)$ and $B^-\to(K^0X_s,K^{*0}X_s)$ are the most promising ones in searching for direct CP violation; they have branching ratios of order $10^{-6}-10^{-4}$ and CP rate asymmetries of order $(10-40)\%$. Note that as shown in Eqs. (3.21) and (3.22), a measurement of partial rate difference of $\overline{B}^0_s\to(\pi^0,\rho^0,\omega)X_{\bar{s}}$ and $B^-\to(K^0X_s,K^{*0}X_s)$ will provide useful information on the unitarity angle α, while $\overline{B}^0_s\to\rho^0X_{s\bar{s}}$ and $\overline{B}^0\to(K^-X,K^{*-}X)$ on the angle γ. To have a rough estimate of the detectability of CP asymmetry, it is useful to calculate the number of $B-\overline{B}$ pairs needed to establish a signal for PRA to the level of three statistical standard deviations given by

$$N_B^{3\sigma}=\frac{9}{\Delta^2 Br\,\epsilon_{\rm eff}}, \tag{4.12}$$

where Δ is the PRA, Br is the branching ratio and $\epsilon_{\rm eff}$ is the product of all of the efficiencies responsible for this signal. With about 1×10^7 $B\overline{B}$ pairs, the asymmetry in K^{*-} channel starts to become accessible; and with about 7×10^7 $B\overline{B}$ events, the PRA's in the other modes mentioned above will become feasible. Here we assumed, for definiteness, $\epsilon_{\rm eff}=1$ and a statistical significance of 3σ as in Eq. (4.12). Currently BaBar has collected 23 million $B\overline{B}$ events, BELLE 11 million pairs and CLEO 9.6 million pairs. It is conceivable that CP asymmetries in semi-inclusive B decays will begin to be accessible at these facilities. Likewise, PRA's in semi-inclusive B_s decays may be measurable in the near future at the Fermilab's Tevatron.

It is interesting to note that the decays $\overline{B}^0_s\to(\pi^0,\rho^0,\omega)X_{s\bar{s}}$ and $B^-\to\phi X$ are electroweak-penguin dominated. Except for the last channel, they have sizable branching

ratios and two of them have observable CP asymmetries. A measurement of these reactions will provide a good probe of electroweak penguins.

Finally, it is useful to discuss briefly the theoretical uncertainties one may have in the present approach for semi-inclusive decays: the b quark mass, the annihilation diagram and the distribution amplitude of the meson. We have assumed a pole mass for the b quark to compute the decay rates of $b \to Mq$ and $B \to MX$ which are proportional to m_b^3. In principle, this uncertainty in m_b can be reduced by normalizing the semi-inclusive hadronic rate to the semi-leptonic one. Since the latter is of 5th power in m_b, the uncertainty is only slightly alleviated. The annihilation topology is power suppressed in the heavy quark limit and is conventionally assumed to be small. However, it is conceivable that power corrections due to the annihilation diagrams could be important for penguin-dominated semi-inclusive decays such as $B^- \to \overline{K}^0(\overline{K}^{*0})X$ and $\overline{B}^0 \to K^-(K^{*-})X$. As for the LCDAs of the meson, we have assumed the asymptotic form (2.16) for the leading-twist LCDA, which is suitable for light mesons but probably not so for the heavy meson J/ψ. Due to SU(3) symmetry breaking, the realistic kaon wave function should exhibit a slight asymmetry in $1-2x$. Also the distribution amplitude of the B meson is not well understood; phenomenologically, the parameter ω_B [see Eq. (4.11)] or λ_B is not well fixed.

V. CONCLUSIONS

We have systematically investigated semi-inclusive B decays $B \to MX$ within the framework of QCD-improved factorization. The nonfactorizable effects, such as vertex-type and penguin-type corrections to the two-body b decay, $b \to Mq$, and hard spectator corrections to the 3-body decay $B \to Mq_1\bar{q}_2$ are calculable in the heavy quark limit. QCD factorization is applicable when the emitted meson is a light meson or a charmonium.

There are two strong phases in the QCD factorization approach: one form final-state rescattering due to hard gluon exchange between M and X, and the other from the penguin diagrams. We have discussed the issue of the CPT constraint on partial rate asymmetries. The strong phase coming from final-state rescattering due to hard gluon exchange between the final states M and X [see Eq. (2.9)] can induce large rate asymmetries for tree-dominated color-suppressed modes $(\pi^0, \rho^0, \omega)X_{\bar{s}}$. The predicted coefficient a_2 in QCD factorization is very small compared to naive factorization. Consequently, the color-suppressed modes $(\pi^0, \rho^0, \omega)X_{\bar{s}}$, ϕX and $J/\psi X_s, J/\psi X$ are very suppressed. Fortunately, the nonfactorizable hard spectator interactions in $B \to Mq_1\bar{q}_2$, though phase-space suppressed, are extremely important for the aforementioned modes. In fact, they are dominated by the hard spectator corrections. This is because the relevant hard spectator correction is color allowed, whereas the two-body semi-inclusive decays for these modes are color-suppressed. Our prediction $\mathcal{B}(B \to J/\psi X_s) = 9.6 \times 10^{-3}$ is in agreement with experiment. Contrary to the exclusive

hadronic decay, the spectator quark corrections here are not subject to the infrared divergent problem, rendering the present prediction more clean and reliable.

Owing to the destructive QCD anomaly effect in the matrix element of pseudoscalar densities for η'-vacuum transition, the four-quark operator contribution to $b \to \eta' s$ is too small to explain the observed fast η' production. It is evident that a new mechanism such as the anomalous coupling of two gluons with the η' is needed in order to resolve the η' puzzle.

The semi-inclusive decay modes: $\overline{B}^0_s \to (\pi^0, \rho^0, \omega) X_{\bar{s}}$, $\rho^0 X_{s\bar{s}}$, $\overline{B}^0 \to (K^- X, K^{*-} X)$ and $B^- \to (K^0 X_s, K^{*0} X_s)$ are the most promising ones in searching for direct CP violation; they have branching ratios of order $10^{-6} - 10^{-4}$ and CP rate asymmetries of order $(10-40)\%$. With about 7×10^7 $B\overline{B}$ pairs, CP asymmetries in these modes may be measurable in the near future at the BaBar, BELLE, CLEO and Tevatron experiments. The decays $\overline{B}^0_s \to (\pi^0, \rho^0, \omega) X_s$ and $B^- \to \phi X$ are electroweak-penguin dominated. Except for the last mode, they in general have sizable branching ratios and two of them have observable CP asymmetries. The above-mentioned reactions will provide good testing ground for the standard model and a good probe for electroweak penguins.

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TABLE I. The coefficients $A^t(\tilde{A}^t)$, $A^p(\tilde{A}^p)$ and B (in units of μ_χ/m_b) defined in Eq. (3.1) for some modes of interest.

Mode	q'	$A^t(\tilde{A}^t)$	$A^p(\tilde{A}^p)$	$B(\mu_\chi/m_b)$
$b\to\pi^-u$	d	a_1	a_4+a_{10}	$2(a_6+a_8)$
$b\to\rho^-u$	d	a_1	a_4+a_{10}	
$b\to\pi^0d$	d	$a_2/\sqrt{2}$	$(-a_4-\frac{3}{2}a_7+\frac{3}{2}a_9+\frac{1}{2}a_{10})/\sqrt{2}$	$3a_8/\sqrt{2}$
$b\to\rho^0d$	d	$a_2/\sqrt{2}$	$(-a_4+\frac{3}{2}a_7+\frac{3}{2}a_9+\frac{1}{2}a_{10})/\sqrt{2}$	
$b\to\omega\, d$	d	$a_2/\sqrt{2}$	$[2a_3+a_4+2a_5+\frac{1}{2}(a_7+a_9-a_{10})]/\sqrt{2}$	
$b\to\phi\, d$	d		$a_3+a_5-\frac{1}{2}(a_7+a_9)$	
$b\to\eta\, d$	d		see text	
$b\to\eta'\, d$	d		see text	
$b\to\pi^-c$	d	a_1		
$b\to\rho^-c$	d	a_1		
$b\to K^0s$	d		$a_4-\frac{1}{2}a_{10}$	$2a_6-a_8$
$b\to K^{*0}s$	d		$a_4-\frac{1}{2}a_{10}$	
$b\to K^-u$	s	a_1	a_4+a_{10}	$2(a_6+a_8)$
$b\to K^{*-}u$	s	a_1	a_4+a_{10}	
$b\to\bar{K}^0d$	s		$a_4-\frac{1}{2}a_{10}$	$2a_6-a_8$
$b\to\bar{K}^{*0}d$	s		$a_4-\frac{1}{2}a_{10}$	
$b\to K^-c$	s	a_1		
$b\to K^{*-}c$	s	a_1		
$b\to\eta\, s$	s		see text	
$b\to\eta'\, s$	s		see text	
$b\to\pi^0s$	s	$a_2/\sqrt{2}$	$\frac{3}{2\sqrt{2}}(-a_7+a_9)$	
$b\to\rho^0s$	s	$a_2/\sqrt{2}$	$\frac{3}{2\sqrt{2}}(a_7+a_9)$	
$b\to\omega\, s$	s	$a_2/\sqrt{2}$	$[2a_3+2a_5+\frac{1}{2}(a_7+a_9)]/\sqrt{2}$	
$b\to\phi\, s$	s		$a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})$	
$b\to J/\psi\, s$	s	a_2	$a_3+a_5+a_7+a_9$	
$b\to J/\psi\, d$	d	a_2	$a_3+a_5+a_7+a_9$	

TABLE II. Numerical values for the coefficients a_i (in units of 10^{-4} for $a_3, \cdots, a_{10}$) in QCD factorization and in naive factorization for $b \to Pq$ and $q' = d, s$.

	QCD factorization		naive factorization
	$q' = d$	$q' = s$	
a_1	$1.05 + 0.01i$	$1.05 + 0.01i$	1.02
a_2	$0.02 - 0.08i$	$0.02 - 0.08i$	0.18
a_3	$74 + 26i$	$74 + 26i$	23
a_4	$-317 - 29i$	$-353 - 58i$	-303
a_5	$-67 - 30i$	$-67 - 30i$	-8
a_6	$-400 - 27i$	$-440 - 45i$	-380
a_7	$-0.29 - 0.65i$	$-0.89 - 1.13i$	0.80
a_8	$3.43 - 0.36i$	$3.22 - 0.46i$	3.89
a_9	$-92.3 - 2.3i$	$-92.9 - 2.8i$	-87.9
a_{10}	$0.8 + 6.6i$	$0.6 + 6.4i$	-12.2

TABLE III. CP-averaged branching ratios and partial-rate asymmetries for some two-body hadronic b decays. For comparison, the predicted branching ratios and rate asymmetries (in absolute values for $\gamma = 60^\circ$) based on naive factorization [4] are given in parentheses.

Mode	BR	PRA(%)
$b \to \pi^- u$	1.5×10^{-4} (1.3×10^{-4})	-2 (7)
$b \to \rho^- u$	4.2×10^{-4} (3.5×10^{-4})	-2 (7)
$b \to \pi^0 d$	5.3×10^{-7} (2.4×10^{-6})	93 (31)
$b \to \rho^0 d$	1.4×10^{-6} (5.9×10^{-6})	91 (33)
$b \to \omega\, d$	2.5×10^{-6} (5.8×10^{-6})	-97 (34)
$b \to \phi\, d$	6.9×10^{-8} (2.3×10^{-7})	-2 (0)
$b \to \pi^- c$	2.2×10^{-2}	0
$b \to \rho^- c$	5.1×10^{-2}	0
$b \to \eta\, d$	1.5×10^{-6}	-59
$b \to \eta' d$	1.0×10^{-6}	38
$b \to K^0 s$	4.0×10^{-6} (2.5×10^{-6})	-20 (4)
$b \to K^{*0} s$	2.6×10^{-6} (2.9×10^{-6})	-24 (14)
$b \to K^- u$	9.2×10^{-5} (2.9×10^{-5})	5 (28)
$b \to K^{*-} u$	4.8×10^{-5} (5.1×10^{-5})	17 (44)
$b \to \bar{K}^0 d$	1.0×10^{-4} (2.0×10^{-5})	0.8 (1)
$b \to \bar{K}^{*0} d$	6.6×10^{-5} (2.6×10^{-5})	0.9 (3)
$b \to K^- c$	1.7×10^{-3}	0
$b \to K^{*-} c$	2.7×10^{-3}	0
$b \to \eta\, s$	1.9×10^{-5}	-4
$b \to \eta' s$	5.4×10^{-5}	1
$b \to \pi^0 s$	1.8×10^{-6} (1.6×10^{-6})	19 (0)
$b \to \rho^0 s$	5.1×10^{-6} (4.3×10^{-6})	19 (0)
$b \to \omega\, s$	3.3×10^{-7} (1.3×10^{-6})	61 (0)
$b \to \phi\, s$	5.5×10^{-5} (6.3×10^{-5})	1 (0)
$b \to J/\psi\, s$	5.4×10^{-4}	-0.5
$b \to J/\psi\, d$	2.8×10^{-5}	10

TABLE IV. CP-averaged branching ratios and partial-rate asymmetries for some semi-inclusive hadronic B decays with $E_M > 2.1$ GeV for light mesons and $E_{J/\psi} > 3.3$ GeV for the J/ψ. Branching ratios due to hard spectator interactions in the 3-body decay $B \to M q_1 \bar{q}_2$ are shown in parentheses. Here X denotes a final state containing no (net) strange or charm particle, and X_q the state containing the quark flavor q.

Mode	BR	PRA(%)
$\overline{B}^0 \to \pi^- X$ ($\overline{B}_s^0 \to \pi^- X_{\bar{s}}$)	1.3×10^{-4} (5.1×10^{-8})	-2
$\overline{B}^0 \to \rho^- X$ ($\overline{B}_s^0 \to \rho^- X_{\bar{s}}$)	3.4×10^{-4} (2.2×10^{-7})	-2
$\overline{B}_s^0 \to \pi^0 X_{\bar{s}}$	1.3×10^{-6} (8.7×10^{-7})	31
$\overline{B}_s^0 \to \rho^0 X_{\bar{s}}$	4.8×10^{-6} (3.7×10^{-6})	22
$\overline{B}_s^0 \to \omega X_{\bar{s}}$	5.5×10^{-6} (3.4×10^{-6})	-37
$B^- \to \phi X$	2.5×10^{-7} (1.9×10^{-7})	-0.5
$\overline{B}^0 \to \pi^- X_c$ ($\overline{B}_s \to \pi^- X_{c\bar{s}}$)	1.8×10^{-2} (8.4×10^{-6})	0
$\overline{B}^0 \to \rho^- X_c$ ($\overline{B}_s \to \rho^- X_{c\bar{s}}$)	4.2×10^{-2} (1.1×10^{-4})	0
$B^- \to K^0 X_s$	3.8×10^{-6} (2.9×10^{-9})	-20
$B^- \to K^{*0} X_s$	2.2×10^{-6} (1.1×10^{-8})	-24
$\overline{B}^0 \to K^- X$ ($\overline{B}_s \to K^- X_{\bar{s}}$)	8.7×10^{-5} (3.6×10^{-9})	5
$\overline{B}^0 \to K^{*-} X$ ($\overline{B}_s \to K^{*-} X_{\bar{s}}$)	3.9×10^{-5} (1.4×10^{-8})	16
$B^- \to \overline{K}^0 X$	9.7×10^{-5} (7.5×10^{-8})	0.8
$B^- \to \overline{K}^{*0} X$	5.4×10^{-5} (2.9×10^{-7})	0.9
$\overline{B}^0 \to K^- X_c$ ($\overline{B}_s^0 \to K^- X_{c\bar{s}}$)	1.4×10^{-3} (4.3×10^{-7})	0
$\overline{B}^0 \to K^{*-} X_c$ ($\overline{B}_s^0 \to K^{*-} X_{c\bar{s}}$)	2.3×10^{-3} (4.8×10^{-6})	0
$\overline{B}_s^0 \to \pi^0 X_{s\bar{s}}$	1.5×10^{-6} (5.0×10^{-8})	19
$\overline{B}_s^0 \to \rho^0 X_{s\bar{s}}$	4.4×10^{-6} (2.2×10^{-7})	18
$\overline{B}_s^0 \to \omega X_{s\bar{s}}$	7.4×10^{-6} (7.1×10^{-6})	2
$B^- \to \phi X_s$	5.8×10^{-5} (2.8×10^{-6})	1
$B \to J/\psi X_s$	9.6×10^{-3} (9.2×10^{-3})	0
$B \to J/\psi X$	5.1×10^{-4} (4.9×10^{-4})	0.5